

Remark About dS/CFT Correspondence

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ABSTRACT: In this paper we will study some aspects of dS/CFT correspondence. We will focus on the relation between Witten's non-standard de Sitter inner product and correlators in the holographic dual conformal field theory. We will argue that from the definition of Witten's inner product and conjecture that the Hilbert space of initial states of massive scalar field on \mathcal{I}^- in de Sitter space corresponds to the Hilbert space of states of Euclidean CFT on \mathcal{I}^- , we can obtain CFT correlators in any vacuum state.

KEYWORDS: dS/CFT, string theory.

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Contents

1. Introduction

Remarkable series of recent observations suggest that we live in accelerating space-time [1, 2, 3, 4, 5]. However understanding the quantum theory of gravity in de Sitter space remains one of the most important problems in theoretical physics.¹ A correspondence relating de Sitter (dS) gravity to the conformal field theory (CFT) theory dual was presented in [13] and further studied in [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]. For further details considering this proposal, see [30, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40].

The dS/CFT correspondence is mainly modeled with analogy AdS/CFT correspondence [42, 43, 44]: Like the AdS case, the symmetries of de Sitter space suggest that the dual theory is conformally invariant. Mainly, it was proposed the connection between gravity in de Sitter space and the correlation functions in the dual Euclidean CFT. It is also clear that the nature of de Sitter space is different from its AdS counterpart. For example, the conformal boundaries of dS are hyper-surfaces of Euclidean signature. Further, in de Sitter space, there are two such hyper-surfaces, the future boundary \mathcal{I}^+ and the past boundary \mathcal{I}^- . There was great debate in the past few months if the proposed duality involves single CFT [13] living on either \mathcal{I}^+ or \mathcal{I}^- , or if we should rather consider both of them as independent parts of one single theory and study nontrivial correlation between them, as was suggested in very interesting paper [35]. This question was addressed in [38] where very impressive arguments were given that support the idea of single dual CFT. In this paper we will consider the CFT living on \mathcal{I}^- as the fundamental one, with agreement with [13]. We will also sometimes consider operators in the second CFT living on \mathcal{I}^+ however as will be clear from the context the operators in CFT on \mathcal{I}^+ are determined by operators of CFT on \mathcal{I}^- . On the other hand it is possible that the nontrivial relation between CFT living on $\mathcal{I}^+, \mathcal{I}^-$ could capture dynamic of bulk quantum gravity.

However, the most striking difference from the AdS/CFT duality is the fact that we have no rigorous realization of the dS/CFT correspondence. In [13, 30] dS/CFT duality was proposed on the direct analogy with AdS/CFT correspondence when the

¹Very exciting and radical proposals considering string theory in de Sitter space can be found in [6, 7, 8]. Alternative point of view on the quantum gravity in three dimensional de Sitter space can be found in [9]. For recent discussion of some issues of quantum theory in de Sitter space, see [10, 11, 12].

correlation functions in the dual CFT theory are determined through the bulk S-matrix elements of massive scalar field. However it is important to stress that there is no rigorous definition of such a duality as opposite with Ads/CFT correspondence.

In [41] different point of view on the problem of quantum gravity in dS space and its possible connection to the dual CFT was suggested. It was stressed by author that his approach seems to give increasingly less information as the cosmological constant is increased. More precisely, Witten argued that for negative cosmological constant the sort of reasoning, that he presented there, gives the boundary conformal field theory which, according to [41] might be regarded as supplying a dynamical principle. For zero cosmological constant we get S-matrix. For positive cosmological constant all what we get is definition of the Hilbert space \mathcal{H}_i of initial states at \mathcal{I}^- or equivalently the Hilbert space \mathcal{H}_f of final states at \mathcal{I}^+ . In [41] also definition of the Hilbert spaces $\mathcal{H}_{i,f}$ and inner product was suggested. The modified inner product involves path integral evolution from \mathcal{I}^- to \mathcal{I}^+ together with \mathcal{CPT} conjugation. The concrete realization of this proposal was given in [29] in the case of the free massive scalar field in dS_3 . It was argued that the inner product for the bulk scalar field constructed along the lines given in [41] leads to the adjoint in the dual CFT that gives standard inner product of the boundary field theory.

In this paper we would like to apply these ideas and argue that from the proposed modified inner product and the relation between CFT living on \mathcal{I}^- and the bulk scalar field we can extract all correlation functions in the dual CFT without any explicit form of the conjectured dS/CFT duality. We begin with the review of the construction of the inner product in the space of initial states of the free massive field in dS_3 as was given in [29] that leads to the definition of the Hilbert space \mathcal{H}_i of the initial states of the scalar field on \mathcal{I}^- . According to the dS/CFT duality states in \mathcal{H}_i should correspond to the states of Euclidean CFT living on \mathcal{I}_- . Since the construction of the inner product presented in [29] is given in terms of $|in\rangle$ vacua with the natural particle interpretation, it is reasonable to presume an existence of dual CFT operator \mathcal{O}_+^{in} that creates these quanta from the vacuum state $|in\rangle$. From the same reason we can deduce an existence of lowering operator \mathcal{O}_-^{in} and also that these operators must obey canonical commutation relations. From these commutation relations and definition of the inner product we will be able to reproduce all two point functions in $|in\rangle$ vacuum state as were given in [29, 30] as well as the correlation functions for operators defined on \mathcal{I}^+ and \mathcal{I}^- , even it is clear that in case of the free massive scalar field in the bulk the CFT operators on \mathcal{I}^+ are uniquely functions of the CFT operators on \mathcal{I}^- . It is also well known that there is one parameter family of vacuum states ² for free massive scalar field in dS. It is then natural to presume that these vacuum states should have their description in CFT as

²The quantum field theory of massive scalar field in de Sitter space time was extensively studied in the past, see for example [45, 46, 47, 48, 49]. For general introduction to the quantum field theory in curved space time, see [50, 51, 52]. There was great discussion about consistency of general vacuum states $|\gamma\rangle$ and their possible meaning in cosmology, see for example [54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65].

well. In fact, CFT correlators in general vacuum states $|\gamma\rangle$ were previously calculated in [29, 30]. We extend our analysis to general vacuum states as well and we will show that we are able to reproduce results given in [29, 30].

We hope that our calculation could be helpful for better understanding of the proposed dS/CFT correspondence. The main point of our calculations is the conjecture that the dual CFT gives an exact description of the Hilbert space on \mathcal{I}^- . This approach is mainly motivated by Witten's arguments that dual CFT theory can only give an information about the structure of the Hilbert space of quantum gravity of initial states on \mathcal{I}^- rather to supplies for dynamical principle as in the case of AdS/CFT correspondence. It is possible that this dynamical principle is somewhat hidden in the relation between CFT operators on \mathcal{I}^- and the operators living on \mathcal{I}^+ and that this relation could support the dynamical evolution of the bulk quantum gravity.

It seems to us that the possibility to extract CFT correlation functions directly from the definition of the inner product without any explicit realization of the dS/CFT correspondence could be helpful for further study of the bulk quantum theory. For example, there is Schrödinger picture description of the quantum field in dS space (See, for example [66] and reference there.) and at present it is not complete clear to us how this description could have its holographic description in dual CFT. We hope that our approach could be useful for addressing this issue.

The organization of this paper is follows. In section 2 we review the quantisation of the massive scalar field in dS space. In section 3 we review the construction of the inner product for the bulk scalar field as was given in [29] and we obtain all two point functions in the dual CFT in $|in\rangle$ vacuum state. In section 4 we will discuss these correlators in general vacuum states $|\gamma\rangle$. In conclusion 5 we summarize our results and suggest further direction of research.

2. Massive scalar field in de Sitter space

In this section we review the quantisation of the massive scalar field in de Sitter space following mainly very nice analysis given in [29].

d-dimensional de Sitter space (dS_d) is described by hyperboloid in $d+1$ dimensional Minkowski space ³

$$P(X, X) = 1, P(X, Y) = \eta_{ab} X^a X^b, a, b = 0, \dots, d. \quad (2.1)$$

In convention [29] lower case x denote d -dimensional coordinate on dS_d and upper case X to denote the corresponding $d+1$ dimensional coordinate in the embedding space.

There are many coordinate systems that are useful for description of dS_d , for recent review, see [52, 53]. In this paper we will work in the global coordinates (τ, Ω) in which de Sitter metric is

$$ds^2 = -d\tau^2 + \cosh^2 \tau d\Omega_{d-1}^2, \quad (2.2)$$

³With the metric $\eta_{ab} = \text{diag}(-1, 1, \dots, 1)$.

where $d\Omega_{d-1}^2$ is the standard round metric on S^{d-1} . The equation of motion of the scalar field is

$$(\nabla^2 - m^2)\phi = 0, \quad \nabla^2\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi) \quad (2.3)$$

that in the metric (2.2) has the form

$$-\frac{1}{\cosh^{d-1}\tau}\partial_\tau(\cosh^{d-1}\tau\partial_\tau\phi) + \frac{1}{\cosh^d\tau}\nabla_{S^{d-1}}^2\phi - m^2\phi = 0. \quad (2.4)$$

This equation is separable with the solution

$$\phi = y_L(\tau)Y_{Lj}(\Omega), \quad (2.5)$$

where Y_{Lj} are spherical harmonic on S^{d-1} obeying

$$\nabla_{S^{d-1}}^2 Y_{Lj} = -L(L+d-2)Y_{Lj}, \quad (2.6)$$

where L is non-negative integer and j is a collective index (j_1, \dots, j_{d-2}) . Following [29] we choose such Y_{Lj} 's that obey

$$Y_{Lj}(\Omega_A) = Y_{Lj}^*(\Omega) = (-1)^L Y_{Lj}(\Omega), \quad (2.7)$$

where Ω_A denotes the point on S^{d-1} antipodal to Ω . The functions Y_{Lj} are orthonormal

$$\int d\Omega Y_{Lj}(\Omega) Y_{L'j'}^*(\Omega) = \delta_{LL'} \delta_{jj'}, \quad (2.8)$$

and complete

$$\sum_{Lj} Y_{Lj}(\Omega) Y_{Lj}^*(\Omega') = \delta^{d-1}(\Omega, \Omega'). \quad (2.9)$$

For real positive μ , $2m > (d-1)$ we find ⁴

$$y_L^{in} = \frac{2^{L+d/2-1}}{\sqrt{\mu}} \cosh^L \tau e^{(L+\frac{d-1}{2}-i\mu)\tau} F\left(L + \frac{d-1}{2}, L + \frac{d-1}{2} - i\mu, 1 - i\mu, -e^{2\tau}\right) \quad (2.10)$$

which together with its complex conjugate are two independent solutions. The normalization is fixed that these modes are orthonormal with respect to the standard inner product

$$(\phi_{Lj}, \phi_{L'j'}) = -i \int d\Omega \cosh^{d-1} \tau \left(\partial_\tau \phi_{Lj}^* \phi_{L'j'} - \phi_{Lj}^* \partial_\tau \phi_{L'j'} \right) = \delta_{LL'} \delta_{jj'}, \quad (2.11)$$

where the integral is performed over S^{d-1} at fixed τ . As is well known (See, for example [50].) this inner product is independent on time slicing for modes obeying (2.3) so that we can evaluate it in the limit $\tau \rightarrow -\infty$ when $F \rightarrow 1$, $\cosh \tau \rightarrow \frac{1}{2}e^{-\tau}$ and consequently

$$\lim_{\tau \rightarrow -\infty} \phi_{Lj}^{in}(x) = \frac{2^{d/2-1}}{\sqrt{\mu}} e^{(\frac{d-1}{2}-i\mu)\tau} Y_{Lj}(\Omega). \quad (2.12)$$

⁴ $\mu = \sqrt{m^2 - \frac{(d-1)^2}{4}}$. In this paper we will consider μ real positive only. For aspect of the quantisation of the scalar fields with other values of μ , see recent papers [31, 34] and reference therein.

Then it is simple task to show validity (2.11).

From (2.12) it is clear that ϕ^{in} modes correspond to the positive frequency modes with respect to the global time τ near the asymptotic past and hence they represent incoming particle states. Then we define $|in\rangle$ vacuum as a state which is annihilated by the lowering operator associated to ϕ^{in} . Physically, $|in\rangle$ is the state with no incoming particles on \mathcal{I}^- .

We also see that (2.4) is invariant under time reversal $\tau \rightarrow -\tau$. Hence we can obtain another pair of linearly independent solutions by defining

$$y_L^{out}(\tau) = y_L^{in*}(-\tau) \quad (2.13)$$

with the asymptotic behavior

$$\lim_{\tau \rightarrow \infty} y^{out} \rightarrow \frac{2^{d/2-1}}{\sqrt{\mu}} e^{-(\frac{d-1}{2} + i\mu)\tau} \quad (2.14)$$

which implies that modes

$$\phi_{Lj}^{out}(x) = y_L^{out}(\tau) Y_{Lj}(\Omega) \quad (2.15)$$

are positive frequency modes with respect to the global time τ near the asymptotic future and represent outgoing particle states. As in the case of ϕ_{Lj}^{in} we can define vacuum state $|out\rangle$ which is the state annihilated by the lowering operators associated to ϕ^{out} . Generally $|in\rangle$ state is not the same as $|out\rangle$ state however as was recently reviewed in [29], for odd dimensional de Sitter space we can identify them

$$|in\rangle = |out\rangle \text{ in odd dimensions .} \quad (2.16)$$

It follows that there is no particle production in the sense that if no particles are coming in from \mathcal{I}^- no particles will go out on \mathcal{I}^+ . This is in contrast to the even-dimensional case for which there is always some particle production.

Note that (2.12) implies asymptotic behavior of ϕ^{in} near \mathcal{I}^-

$$\phi_{\pm}^{in} \sim e^{h_{\pm}\tau} , h_{\pm} = \frac{d-1}{2} \pm i\mu , \text{ for } \tau \rightarrow -\infty . \quad (2.17)$$

As was conjectured in [13] ϕ_-^{in} is dual to the operators of weight h^+ in the CFT living on \mathcal{I}^- . In the same way ϕ_+^{in} is dual to operator of conformal weight h_- .

As was shown in [29] it is useful for the definition of general vacuum states $|\gamma\rangle$ to introduce rescaled global modes

$$\begin{aligned} \tilde{\phi}_{Lj}^{in}(x) &= e^{i\theta_L} y_L^{in}(\tau) Y_{Lj}(\Omega) , \\ \tilde{\phi}_{Lj}^{out}(x) &= e^{-i\theta_L} y_L^{out}(\tau) Y_{Lj}(\Omega) , \end{aligned} \quad (2.18)$$

where the phase $e^{i\theta_L}$ is defined

$$e^{-2i\theta_L} = (-1)^{L-\frac{d-1}{2}} \frac{\Gamma(-i\mu)\Gamma\left(L + \frac{d-1}{2} + i\mu\right)}{\Gamma(i\mu)\Gamma\left(L + \frac{d-1}{2} - i\mu\right)} . \quad (2.19)$$

These rescaled modes will be used in the discussion of the general vacuum state in section 4.

In the following we will also need to know how at the general point in the bulk the field is determined from its value on \mathcal{I}^- . It can be shown that these fields are related through

$$\begin{aligned}\phi(x) &= i \int_{\mathcal{I}^-} d\Omega' \sqrt{g(\tau')} (\partial_{\tau'} G_C(x, x') \phi(x') - G_C(x, x') \partial_{\tau'} \phi(x')) , \\ G_C(x, x') &= G_E(x, x') - G_E(x', x) , G_E(x, x') = \langle E | \phi(x) \phi(x') | E \rangle ,\end{aligned}\tag{2.20}$$

where $|E\rangle$ is Euclidean vacuum and $G_E(x, x')$ is Wightman function characterized given vacuum state, for more details and very nice discussion, see again [29]. It is important to stress that G_c does not depend on vacuum state that defines Wightmann function. For our purposes it is important to find limiting value of $G_C(x, x')$ for $\tau \rightarrow \infty$. It can be shown that [29]

$$\begin{aligned}\lim_{\tau' \rightarrow -\infty, \tau \rightarrow \infty} G_E(x, x') &= -e^{h_+(\tau' - \tau)} e^{-\pi\mu} \Delta_+(\Omega', \Omega_A) - e^{h_-(\tau' - \tau)} e^{\pi\mu} \Delta_-(\Omega', \Omega_A) , \\ \lim_{\tau' \rightarrow -\infty, \tau \rightarrow \infty} G_E(x', x) &= -e^{h_+(\tau' - \tau)} e^{\pi\mu} \Delta_+(\Omega', \Omega_A) - e^{h_-(\tau' - \tau)} e^{-\pi\mu} \Delta_-(\Omega', \Omega_A) ,\end{aligned}\tag{2.21}$$

where

$$\Delta_{\pm}(\Omega, \Omega') = -\frac{1}{\mu \sinh \pi\mu} \sum_{Lj} e^{\mp 2i\theta_L} Y_{Lj}(\Omega) Y_{Lj}(\Omega')\tag{2.22}$$

are two point functions for a conformal field of dimension h_{\pm} on the sphere. Then we get

$$\lim_{\tau' \rightarrow -\infty, \tau \rightarrow \infty} G_C(x, x') = 2e^{h_+(\tau' - \tau)} \sinh \pi\mu \Delta_+(\Omega', \Omega_A) - 2 \sinh \pi\mu e^{h_-(\tau' - \tau)} \Delta_-(\Omega', \Omega_A) .\tag{2.23}$$

Let us introduce $\phi_{\pm}^{in}(\Omega)$, $\phi_{\pm}^{out}(\Omega)$ fields on \mathcal{I}^- , \mathcal{I}^+ by

$$\begin{aligned}\lim_{\tau \rightarrow -\infty} \phi(\tau, \Omega) &= \phi_+^{in}(\Omega) e^{h_+\tau} + \phi_-^{in}(\Omega) \phi_-^{h-\tau} , \\ \lim_{\tau \rightarrow \infty} \phi(\tau, \Omega_A) &= \phi_+^{out}(\Omega) e^{h_+\tau} + \phi_-^{out}(\Omega) \phi_-^{h-\tau} ,\end{aligned}\tag{2.24}$$

where, according to [13, 29] ϕ_{\pm}^{out} have been defined with an antipodal inversion relative to ϕ_{\pm}^{in} . Asymptotic form (2.12) for $d = 3$ implies

$$\begin{aligned}\phi_+^{in}(\Omega) &= \sqrt{\frac{2}{\mu}} \sum_{Lj} b_{Lj}^{in} Y_{Lj}^*(\Omega) , \phi_-^{in}(\Omega) = \sqrt{\frac{2}{\mu}} \sum_{Lj} a_{Lj}^{in} Y_{Lj}(\Omega) \\ \phi_+^{out}(\Omega) &= \sqrt{\frac{2}{\mu}} \sum_{Lj} a_{Lj}^{out} Y_{Lj}(\Omega_A) , \phi_-^{out}(\Omega) = \sqrt{\frac{2}{\mu}} \sum_{Lj} b_{Lj}^{out} Y_{Lj}^*(\Omega_A) .\end{aligned}\tag{2.25}$$

Then (2.20) gives

$$\phi_{\pm}^{out}(\Omega) = -\mu \sinh \pi \mu \int d\Omega \Delta_{\pm}(\Omega, \Omega') \phi_{\mp}^{in}(\Omega') . \quad (2.26)$$

In the next section we use many of the facts reviewed above for the construction of the non-standard inner product [41] and its realization in the case of free massive scalar field in [29].

3. CFT correlators from inner product

In [41] Witten suggested how to construct Hilbert spaces of initial and final states of quantum gravity in de Sitter space, together with the definition of inner product. This definition is based on the path integral evolution from \mathcal{I}^- to \mathcal{I}^+ together with \mathcal{CPT} conjugation. Since we believe that there is an exact description of the quantum gravity and quantum field theory in de Sitter space in terms of dual CFT living on \mathcal{I}^- we expect that the Hilbert space of the initial states of scalar field in the bulk is described using CFT operators acting on some vacuum state. We will show that from the definition of the inner product according to Witten and its very nice analysis in [29] we can obtain all correlation functions in the dual CFT.

To begin with, we briefly review discrete symmetries \mathcal{C} , \mathcal{P} and \mathcal{T} in de Sitter space, following [29]. As in this paper we restrict ourselves to the three dimensional de Sitter space.

Let us consider the real scalar field so that the operation of charge conjugation \mathcal{C} is trivial. We define action of the following discrete symmetry operations on this field

$$\mathcal{P}\phi(x)\mathcal{P} = \phi(Px) , \mathcal{T}\phi(x)\mathcal{T} = \phi(Tx) , \quad (3.1)$$

where we have two discrete symmetries of de Sitter space P and T that are defined as [29]

$$\begin{aligned} PX^0 &= X^0 , PX^1 = X^1 , PX^2 = -X^2 , PX^3 = -X^3 , \\ TX^0 &= -X^0 , TX^1 = X^1 , TX^2 = X^2 , TX^3 = X^3 \end{aligned} \quad (3.2)$$

that clearly leave (2.1) invariant. Bulk scalar field ϕ can be written as

$$\begin{aligned} \phi(\tau, \Omega) &= \sum_{L,j} \left(a_{Lj}^{in} y_L^{in}(\tau) Y_{Lj}(\Omega) + b_{Lj}^{in} y_L^{in*}(\tau) Y_{Lj}^*(\Omega) \right) , \\ \phi(\tau, \Omega) &= \sum_{L,j} \left(a_{Lj}^{out} y_L^{out}(\tau) Y_{Lj}(\Omega) + b_{Lj}^{out} y_L^{out*}(\tau) Y_{Lj}^*(\Omega) \right) . \end{aligned} \quad (3.3)$$

As in [29] we have introduced lowering and raising operators a_{Lj}, b_{Lj} respectively and are not assuming that $a_{Lj}^\dagger = b_{Lj}$. Then the action of the discrete symmetries \mathcal{P}, \mathcal{T} was

defined in such a way that their definition reproduces (3.1). More precisely we have

$$\mathcal{P}a_{Lj}^{in}\mathcal{P} = (-1)^j a_{Lj}^{in}, \mathcal{P}b_{Lj}^{in}\mathcal{P} = (-1)^j b_{Lj}^{in}. \quad (3.4)$$

Since $Y_{Lj}(P\Omega) = (-1)^j Y_{Lj}(\Omega)$ we immediately see that this definition reproduces the first line in (3.1). In the same way

$$\mathcal{T}a_{Lj}^{in}\mathcal{T} = (-1)^L a_{Lj}^{in}, \mathcal{T}b_{Lj}^{in}\mathcal{T} = (-1)^L b_{Lj}^{in}. \quad (3.5)$$

We note that time reversal transformation \mathcal{T} is anti linear operator which combines unitary operator U with complex conjugation K of the functions. Then it can be shown [29]

$$\begin{aligned} \mathcal{PT}\phi_+^{in}(\Omega)\mathcal{PT} &= \phi_-^{out}(P\Omega_A), \\ \mathcal{PT}\phi_+^{out}(\Omega)\mathcal{PT} &= \phi_-^{in}(P\Omega_A). \end{aligned} \quad (3.6)$$

Now we will review the construction of the Witten's inner product [41] given in [29]. Let us consider one particle states on \mathcal{I}^\pm

$$\begin{aligned} |\Psi^{in}\rangle &= \int d\Omega \Psi^{in}(\Omega) \phi_+^{in}(\Omega) |in\rangle, \\ |\Psi^{out}\rangle &= \int d\Omega \Psi^{out}(\Omega) \phi_-^{out}(\Omega) |out\rangle. \end{aligned} \quad (3.7)$$

Using (2.26) we can express $|\Psi^{out}\rangle$ as

$$|\Psi^{out}\rangle = -\mu \sinh \pi\mu \int d\Omega d\Omega' \Psi^{out}(\Omega) \Delta_-(\Omega, \Omega') \phi_+^{in}(\Omega') |in\rangle. \quad (3.8)$$

This defines the bilinear pairing

$$(\Psi^{out}|\Psi^{in}) = -\mu \sinh \pi\mu \int d\Omega d\Omega' \Psi^{out}(\Omega') \Delta_-(\Omega', \Omega) \Psi^{in}(\Omega). \quad (3.9)$$

Now we apply the \mathcal{CPT} conjugation to any state on \mathcal{I}^- to give a state on \mathcal{I}^+

$$\mathcal{CPT}|\Psi^{in}\rangle = \int d\Omega \Psi^{*in}(\Omega) \phi_-^{out}(P\Omega_A) |in\rangle = \int d\Omega \Psi^{in*}(P\Omega_A) \phi_-^{out}(\Omega) |in\rangle. \quad (3.10)$$

Then we can use the previous pairing to define the inner product on \mathcal{I}^-

$$\langle \Psi^{in} | \Gamma^{out} \rangle = -\mu \sinh \pi\mu \int d\Omega d\Omega' \Psi^{in*}(\Omega) \Delta_-(P\Omega_A, \Omega') \Gamma^{in}(\Omega'). \quad (3.11)$$

According to dS/CFT conjecture any state in the Hilbert space \mathcal{H}_i should have description in the dual CFT. Using well known CFT state-operator correspondence we expect that this state is given by action of some operator \mathcal{O}_+^{in} on the vacuum state $|in\rangle$. Let us denote the complete set of these operators as \mathcal{O}_{+Lj}^{in} where the quantum numbers

L, j have the same meaning as in section 2. Then any state in the Hilbert space \mathcal{H}_i can be written as

$$|\Gamma\rangle = \sum_{L,j} f_{Lj} \mathcal{O}_{+Lj}^{in} |in\rangle = \sqrt{\frac{2}{\mu}} \int d\Omega \Psi^{in}(\Omega) \mathcal{O}_+^{in}(\Omega) |in\rangle \quad (3.12)$$

using

$$\mathcal{O}_+^{in}(\Omega) = \sqrt{\frac{\mu}{2}} \sum_{Lj} Y_{Lj}^{in*}(\Omega) \mathcal{O}_{+Lj}^{in}, \quad \Psi^{in}(\Omega) = \sum_{Lj} f_{Lj} Y_{Lj}^{in}(\Omega). \quad (3.13)$$

We define \mathcal{O}_{+Lj}^{in} as creation operator in CFT Hilbert space \mathcal{H}_i . It is then natural to postulate an existence of the annihilation operator \mathcal{O}_{-Lj}^{in} with the canonical commutation relation

$$[\mathcal{O}_{-Lj}^{in}, \mathcal{O}_{+L'j'}^{in}] = \delta_{LL'} \delta_{jj'} \quad (3.14)$$

which implies

$$\begin{aligned} [\mathcal{O}_-^{in}(\Omega), \mathcal{O}_+^{in}(\Omega')] &= \left[\sqrt{\frac{\mu}{2}} \sum_{Lj} Y_{Lj}^*(\Omega) \mathcal{O}_{-Lj}^{in}, \sqrt{\frac{\mu}{2}} \sum_{L'j'} Y_{L'j'}(\Omega') \mathcal{O}_{+L'j'}^{in} \right] = \\ &= \frac{\mu}{2} \sum_{Lj, L'j'} Y_{Lj}^*(\Omega) Y_{L'j'}(\Omega') \delta_{LL'} \delta_{jj'} = \frac{\mu}{2} \delta(\Omega, \Omega'). \end{aligned} \quad (3.15)$$

From the upper expression we immediately get two point function (Note that the commutator is pure number and hence its vacuum expectation value is the same for all vacuum states.)

$$\langle in | [\mathcal{O}_-^{in}(\Omega), \mathcal{O}_+^{in}(\Omega')] | in \rangle = \frac{\mu}{2} \delta(\Omega, \Omega') \Rightarrow \langle in | \mathcal{O}_-^{in}(\Omega) \mathcal{O}_+^{in}(\Omega') | in \rangle = \frac{\mu}{2} \delta(\Omega, \Omega') \quad (3.16)$$

using

$$\mathcal{O}_-^{in}(\Omega) |in\rangle = 0. \quad (3.17)$$

In order to determine other two point functions we use the definition of the inner product given above. This is the main point of our analysis that should deserve deeper explanation. In the usual dS/CFT correspondence the CFT correlators are determined from the possible equivalence of CFT partition function and S-matrix in the bulk, which is the proposal given in [13, 30]. The motivation for this proposal comes from the AdS/CFT correspondence. However when we adopt Witten's arguments [41] considering the dependence of the holographic description of the quantum gravity on cosmological constant and his conclusion that CFT could give information about Hilbert space of initial states \mathcal{H}_i it seems to us very natural to search for dS/CFT correspondence in the structure of this Hilbert space and in particular in the definition of the inner product in \mathcal{H}_i . We will see that this analysis really allows us to get desired two point functions. In the CFT the inner product is defined as

$$\langle \Psi^{in} | \Gamma^{out} \rangle = \frac{2}{\mu} \int d\Omega d\Omega' \Psi^{in*}(\Omega) \langle (\mathcal{O}_+^{in}(\Omega))^\dagger \mathcal{O}_+^{in}(\Omega') \rangle \Gamma^{in}(\Omega') \quad (3.18)$$

so that comparing this expression with (3.11) gives

$$\frac{2}{\mu} \langle (\mathcal{O}_+^{in}(\Omega))^\dagger \mathcal{O}_+^{in}(\Omega') \rangle = -\mu \sinh \pi \mu \Delta_-(P\Omega_A, \Omega') . \quad (3.19)$$

Even if we do not know the precise form of the dS/CFT correspondence we can at least try to suggest the interaction between CFT operators and the bulk field. We expect that the interaction between bulk fields and boundary CFT operators has following local form

$$\begin{aligned} & \int d\Omega \left(\mathcal{O}_+^{in}(\Omega) \phi_+^{in}(\Omega) + \mathcal{O}_-^{in}(\Omega) \phi_-^{in}(\Omega) \right) , \\ & \int d\Omega \left(\mathcal{O}_+^{out}(\Omega) \phi_+^{out}(\Omega) + \mathcal{O}_-^{out}(\Omega) \phi_-^{out}(\Omega) \right) \end{aligned} \quad (3.20)$$

where we now consider the second CFT living on \mathcal{I}^+ even if it will be clear in the moment that there is unique relation between CFT operators $\mathcal{O}^{in}, \mathcal{O}^{out}$ at least in the case of the free massive field in the bulk. Now when we apply (2.26) in (3.20) we obtain

$$\begin{aligned} & \int d\Omega \left(\mathcal{O}_+^{in}(\Omega) \phi_+^{in}(\Omega) + \mathcal{O}_-^{in}(\Omega) \phi_-^{in}(\Omega) \right) = \\ & = -\mu \sinh \pi \mu \int d\Omega d\Omega' \left(\mathcal{O}_+^{in}(\Omega) \Delta_+(\Omega, \Omega') \phi_-^{out}(\Omega') + \right. \\ & \quad \left. + \mathcal{O}_-^{in}(\Omega) \Delta_-(\Omega, \Omega') \phi_+^{out}(\Omega') \right) \end{aligned} \quad (3.21)$$

so that when we compare this expression with the second line in (3.20) we obtain the relation between *in* and *out* CFT operators

$$\mathcal{O}_\mp^{out} = -\mu \sinh \pi \mu \int d\Omega' \Delta_\pm(\Omega, \Omega') \mathcal{O}_\pm^{in}(\Omega') \quad (3.22)$$

with agreement with [30]. We see that it is natural to define the adjoint operation in the CFT as

$$(\mathcal{O}_\pm^{in}(\Omega))^\dagger = \mathcal{O}_\pm^{out}(P\Omega_A) , (\mathcal{O}_\pm^{out}(\Omega))^\dagger = \mathcal{O}_\pm^{in}(P\Omega_A) \quad (3.23)$$

since now (3.19) is equal to

$$\begin{aligned} & \frac{2}{\mu} \langle (\mathcal{O}_+^{in}(\Omega))^\dagger \mathcal{O}_+^{in}(\Omega') \rangle = \frac{2}{\mu} \langle \mathcal{O}_+^{out}(P\Omega_A) \mathcal{O}_+^{in}(\Omega') \rangle = \\ & = -2 \sinh \pi \mu \int d\Omega'' \Delta_-(P\Omega_A, \Omega'') \langle \mathcal{O}_-^{in}(\Omega'') \mathcal{O}_+^{in}(\Omega') \rangle = \\ & = -2 \sinh \pi \mu \int d\Omega'' \Delta_-(P\Omega_A, \Omega'') \frac{\mu}{2} \delta(\Omega'', \Omega') = -\mu \sinh \pi \mu \Delta_-(P\Omega_A, \Omega') \end{aligned} \quad (3.24)$$

using (3.22). From the definition of adjoint and the inner product we can get all correlation functions in CFT. First of all, since \mathcal{O}_-^{in} annihilates vacuum state we immediately obtain following correlators

$$\langle \mathcal{O}_-^{in}(\Omega) \mathcal{O}_-^{in}(\Omega') \rangle = 0 , \langle \mathcal{O}_+^{in}(\Omega) \mathcal{O}_-^{in}(\Omega') \rangle = 0 \quad (3.25)$$

with agreement with [29, 30]. We also have

$$\begin{aligned} \langle \mathcal{O}_+^{in}(\Omega) \mathcal{O}_+^{in}(\Omega') \rangle &\sim \int d\Omega'' \Delta_-(\Omega, \Omega'') \langle \mathcal{O}_-^{out}(\Omega'') \mathcal{O}_+^{in}(\Omega') \rangle \sim \\ &\sim \langle (\mathcal{O}_-(P\Omega_A''))^\dagger \mathcal{O}_+^{in}(\Omega') \rangle = 0 , \end{aligned} \quad (3.26)$$

where we used the definition of adjoint given above and the fact that if \mathcal{O}_-^{in} annihilates ket $|in\rangle$ then its adjoint annihilates bra $\langle in|$.

In this section we calculated the two point functions in the CFT defined on \mathcal{I}^- . This calculation was based on the analysis of the modified inner product [29]. Since we began from the vacuum state $|in\rangle$ with its natural particle interpretation we could introduce creation and annihilation operators in CFT that create or annihilate given quanta. Since it is well known that there is one parameter family of different vacuum states of the bulk massive field in dS we would like to see how CFT operators $\mathcal{O}_+^{in}, \mathcal{O}_-^{in}$ act on general vacuum state $|\gamma\rangle$. This analysis will be done in the next section.

4. Correlation functions in γ vacua

In this section we will calculate CFT correlators in other vacuum states $|\gamma\rangle$. Firstly we review the basic facts about different vacuum states of the scalar field in de Sitter space, following mainly [29]. Let us consider the free massive field in de Sitter space with the mode expansion

$$\phi(x) = \sum_n [a_n \phi_n(x) + b_n \phi_n^*(x)] , \quad (4.1)$$

where subscript n labels states of given mode ϕ_n that obeys (2.3). In global coordinates, for example, n means L, j given in the previous section. We have also denoted, following [29] lowering and raising operators as a_n and b_n with commutation relation $[a_n, b_m] = \delta_{nm}$. The Wightman function is defined as

$$G_\gamma(x, x') = \langle \gamma | \phi(x) \phi(x') | \gamma \rangle = \sum_n \phi_n(x) \phi_n^*(x') \quad (4.2)$$

characterized vacuum state $|\gamma\rangle$ that is defined to be the state annihilated by operators associated to positive modes ϕ_n

$$a_n |\gamma\rangle = 0 . \quad (4.3)$$

Since modes ϕ_n satisfy the de Sitter space wave function (2.3), it is easy to see that Wightman function obeys

$$(\nabla^2 - m^2)G_\gamma(x, x') = 0 . \quad (4.4)$$

Let $\tilde{\phi}_n^{in}(x)$ denote the positive frequency modes (2.18) associated to $|in\rangle$ vacuum. Let us consider a new set of modes related by the MA (Mottola-Allen) transform

$$\hat{\phi}_n = N_\gamma (\tilde{\phi}_n^{in} - e^\gamma \tilde{\phi}_n^{in*}) , N_\gamma \equiv \frac{1}{\sqrt{1 - e^{\gamma + \gamma^*}}} ,$$

$$\tilde{\phi}_n = N_\gamma(\hat{\phi}_n + e^\gamma \hat{\phi}_n^\dagger) , \quad (4.5)$$

where γ can be any complex number with $\text{Re}\gamma < 0$. These modes can be used to define new operators \hat{a}_n, \hat{b}_n and hence the vacuum state $|\gamma\rangle$ that is annihilated by operators \hat{a}_n that multiply the positive frequency solutions $\hat{\phi}_n$ in the expansion of the scalar field as

$$\begin{aligned} \phi(x) &= \sum_n \left[\hat{a}_n \hat{\phi}_n(x) + \hat{b}_n \hat{\phi}_n^*(x) \right] = \\ &= \sum_n \left[\tilde{a}_n^{in} \tilde{\phi}_n^{in} + \tilde{b}_n^{in} \tilde{\phi}_n^{in*} \right] \Rightarrow \\ \Rightarrow \hat{a}_n &= N_\gamma(\tilde{a}_n^{in} + e^{\gamma^*} \tilde{b}_n^{in}) , \hat{b}_n = N_\gamma(\tilde{b}_n^{in} + e^\gamma \tilde{a}_n^{in}) . \end{aligned} \quad (4.6)$$

This relation can be written as

$$\hat{a}_n = \mathcal{U} \tilde{a}_n^{in} \mathcal{U}^{-1} , \mathcal{U} = e^K , K = \sum_n (c(\tilde{b}_n^{in})^2 - \bar{c}(\tilde{a}_n^{in})^2) \quad (4.7)$$

with

$$c = -\frac{1}{4} \left(\ln \tanh\left(\frac{\text{Re}\gamma}{2}\right) \right) e^{i\text{Im}\gamma} \equiv A e^{-i\text{Im}\gamma} . \quad (4.8)$$

To prove that \hat{a} can be written as in (4.7) we use following commutation relations

$$[K, \tilde{a}_n^{in}] = \sum_m c[(\tilde{b}_m^{in})^2, \tilde{a}_n^{in}] = -2c\tilde{b}_m^{in} , [K, [\tilde{b}_m^{in}, \tilde{a}_n^{in}]] = -4c\bar{c}\tilde{a}_n^{in} \quad (4.9)$$

so that

$$\begin{aligned} \mathcal{U} \tilde{a}_n^{in} \mathcal{U}^{-1} &= \tilde{a}_n^{in} + \sum_{k=1}^{\infty} \frac{1}{k!} \overbrace{[[K, \tilde{a}_n^{in}], \dots]}^k = (-2c + \frac{8c^2\bar{c}}{6} + \dots) \tilde{b}_n^{in} + \\ &+ (1 - \frac{4c\bar{c}}{2} - \frac{16(c\bar{c})^2}{24} + \dots) \tilde{a}_n^{in} = \sinh(-2A) e^{-i\text{Im}\gamma} \tilde{b}_n^{in} + \cosh(-2A) \tilde{a}_n^{in} . \end{aligned} \quad (4.10)$$

Consequently the vacuum state $|\gamma\rangle$ can be written as

$$|\gamma\rangle = \mathcal{U} |in\rangle . \quad (4.11)$$

We can also express \mathcal{U} using in and out fields as

$$\mathcal{U} = \exp \left(c(\gamma) \frac{\mu}{2} \int d^2\Omega \phi_+^{in}(\Omega) \phi_-^{out}(\Omega) + c(\bar{\gamma}) \frac{\mu}{2} \int d^2\Omega \phi_-^{in}(\Omega) \phi_+^{out}(\Omega) \right) . \quad (4.12)$$

To see this note that in the definition of the MA transformation we used rescaled fields [29]

$$\tilde{\phi}_{Lj}^{in}(x) = e^{i\theta_L} y_L^{in}(\tau) Y_{Lj}(\Omega) . \quad (4.13)$$

This implies that the lowering and raising operators given in the MA transformation formula are related to the operators defined using $\phi_+^{in}(\Omega)$, $\phi_-^{in}(\Omega)$ as $\tilde{b}_n^{in} = e^{i\theta_L} b_n^{in}$, $\tilde{a}_n^{in} = e^{-i\theta_L} b_n^{in}$. Then in the variables corresponding to $\phi_{\pm}^{in,out}(\Omega)$ modes we can express operator K as

$$\begin{aligned} K &= \sum_n (c e^{2i\theta_L} (b_n^{in})^2 - \bar{c} e^{-2i\theta_L} (a_n^{in})^2) = \\ &= c \frac{\mu}{2} \int d\Omega \phi_+^{in}(\Omega) \phi_-^{out}(\Omega) - \bar{c} \frac{\mu}{2} \int d\Omega \phi_-^{in}(\Omega) \phi_+^{out}(\Omega) . \end{aligned} \quad (4.14)$$

According to dS/CFT correspondence we can presume an existence of one parameter family of vacuum states $|\gamma\rangle$ in CFT as well and that these states are given as

$$|\gamma\rangle = \mathcal{U} |in\rangle, \mathcal{U} = e^K, K = c(\gamma) \frac{2}{\mu} \int d\Omega \mathcal{O}_+^{in}(\Omega) \mathcal{O}_-^{out}(\Omega) - \bar{c}(\gamma) \frac{2}{\mu} \int d\Omega \mathcal{O}_-^{in}(\Omega) \mathcal{O}_+^{out}(\Omega) . \quad (4.15)$$

Using (3.23) we can easily show that

$$K^\dagger = -K \quad (4.16)$$

with respect to the adjoint given in the previous section. Then two point functions of CFT operators in general vacuum state $|\gamma\rangle$ are

$$\begin{aligned} \langle \gamma | \mathcal{O}_\pm^{in}(\Omega) \mathcal{O}_\pm^{in}(\Omega') | \gamma \rangle &= \langle e^{-K} \mathcal{O}_\pm(\Omega) e^K e^{-K} \mathcal{O}_\pm^{in}(\Omega') e^K \rangle , \\ \langle \gamma | \mathcal{O}_\pm^{in}(\Omega) \mathcal{O}_\mp^{in}(\Omega') | \gamma \rangle &= \langle e^{-K} \mathcal{O}_\pm^{in}(\Omega) e^K e^{-K} \mathcal{O}_\mp^{in}(\Omega') e^K \rangle . \end{aligned} \quad (4.17)$$

In order to obtain these two point functions we should calculate following expression

$$e^{-K} \mathcal{O}_\pm^{in}(\Omega) e^K = \mathcal{O}_\pm^{in}(\Omega) + \sum_{N=1}^{\infty} \frac{(-1)^N}{N!} \overbrace{[K, [K, \mathcal{O}_\pm^{in}(\Omega)], \dots,]}^N . \quad (4.18)$$

For this calculation we will need following commutators

$$[\mathcal{O}_\pm^{out}(\Omega), \mathcal{O}_\mp^{in}(\Omega')] = -\mu \sinh \pi \mu \int d\Omega'' \Delta_\mp(\Omega, \Omega'') [\mathcal{O}_\mp^{in}(\Omega''), \mathcal{O}_\mp^{in}(\Omega')] = 0 \quad (4.19)$$

and

$$\begin{aligned} [\mathcal{O}_+^{out}(\Omega), \mathcal{O}_+^{in}(\Omega')] &= -\mu \sinh \pi \mu \int d\Omega'' \Delta_-(\Omega, \Omega'') [\mathcal{O}_-^{in}(\Omega''), \mathcal{O}_+^{in}(\Omega')] = \\ &= -\mu \sinh \pi \mu \int d\Omega'' \Delta_-(\Omega, \Omega'') \frac{\mu}{2} \delta(\Omega'', \Omega) = -\frac{\mu^2}{2} \sinh \pi \mu \Delta_-(\Omega, \Omega') , \\ [\mathcal{O}_-^{out}(\Omega), \mathcal{O}_-^{in}(\Omega')] &= -\mu \sinh \pi \mu \int d\Omega'' \Delta_+(\Omega, \Omega'') [\mathcal{O}_+^{in}(\Omega''), \mathcal{O}_-^{in}(\Omega')] = \\ &= \mu \sinh \pi \mu \int d\Omega'' \Delta_+(\Omega, \Omega'') \frac{\mu}{2} \delta(\Omega'', \Omega') = \frac{\mu^2}{2} \sinh \pi \mu \Delta_+(\Omega, \Omega') \end{aligned} \quad (4.20)$$

so we easily get

$$[K, \mathcal{O}_+^{in}(\Omega)] = -2\bar{c}\mathcal{O}_+^{out}(\Omega) , [K, [K, \mathcal{O}_+^{in}(\Omega)]] = 4c\bar{c}\mathcal{O}_+^{in}(\Omega) . \quad (4.21)$$

Then it is easy to see that

$$e^{-K}\mathcal{O}_+^{in}(\Omega)e^K = N_\gamma(\mathcal{O}_+^{in}(\Omega) - e^{\gamma*}\mathcal{O}_+^{out}(\Omega)) \quad (4.22)$$

and consequently

$$\begin{aligned} \langle \gamma | \mathcal{O}_+^{in}(\Omega)\mathcal{O}_+^{in}(\Omega') | \gamma \rangle &= \langle e^{-K}\mathcal{O}_+^{in}(\Omega)e^K e^{-K}\mathcal{O}_+^{in}(\Omega')e^K \rangle = \\ &= N_\gamma^2 \langle (\mathcal{O}_+^{in}(\Omega) - e^{\gamma*}\mathcal{O}_+^{out}(\Omega))(\mathcal{O}_+^{in}(\Omega') - e^{\gamma*}\mathcal{O}_+^{out}(\Omega')) \rangle = \\ &= -N_\gamma^2 e^{\gamma*} \langle \mathcal{O}_+^{out}(\Omega)\mathcal{O}_+^{in}(\Omega') \rangle = N_\gamma^2 e^{\gamma*} \mu \sinh \pi\mu \int d\Omega'' \Delta_-(\Omega, \Omega'') \langle \mathcal{O}_-^{in}(\Omega'')\mathcal{O}_+^{in}(\Omega') \rangle = \\ &= N_\gamma^2 e^{\gamma*} \mu \sinh \pi\mu \int d\Omega'' \Delta_-(\Omega, \Omega'') \frac{\mu^2}{2} \delta(\Omega'', \Omega) = \frac{\mu^2}{2} N_\gamma^2 e^{\gamma*} \sinh \pi\mu \Delta_-(\Omega, \Omega') . \end{aligned} \quad (4.23)$$

Performing the same analysis for \mathcal{O}_-^{in} leads to

$$e^{-K}\mathcal{O}_-^{in}(\Omega)e^K = N_\gamma(\mathcal{O}_-^{in}(\Omega) - e^\gamma\mathcal{O}_-^{out}(\Omega)) \quad (4.24)$$

and hence

$$\begin{aligned} \langle \gamma | \mathcal{O}_-^{in}(\Omega)\mathcal{O}_-^{in}(\Omega') | \gamma \rangle &= \langle e^{-K}\mathcal{O}_-^{in}(\Omega)e^K e^{-K}\mathcal{O}_-^{in}(\Omega')e^K \rangle = \\ &= N_\gamma^2 \langle (\mathcal{O}_-^{in}(\Omega) - e^\gamma\mathcal{O}_-^{out}(\Omega))(\mathcal{O}_-^{in}(\Omega') - e^\gamma\mathcal{O}_-^{out}(\Omega')) \rangle = \\ &= -N_\gamma^2 e^\gamma \langle \mathcal{O}_-^{in}(\Omega)\mathcal{O}_-^{out}(\Omega') \rangle = N_\gamma^2 e^\gamma \mu \sinh \pi\mu \int d\Omega'' \Delta_+(\Omega, \Omega'') \langle \mathcal{O}_-^{in}(\Omega'')\mathcal{O}_+^{in}(\Omega') \rangle = \\ &= N_\gamma^2 e^\gamma \mu \sinh \pi\mu \int d\Omega'' \Delta_+(\Omega, \Omega'') \frac{\mu}{2} \delta(\Omega'', \Omega) = N_\gamma^2 e^\gamma \frac{\mu^2}{2} \sinh \pi\mu \Delta_+(\Omega, \Omega') . \end{aligned} \quad (4.25)$$

Finally we will calculate the mixed correlators

$$\langle \gamma | \mathcal{O}_-^{in}(\Omega)\mathcal{O}_+^{in}(\Omega') | \gamma \rangle = N_\gamma^2 \langle \mathcal{O}_-^{in}(\Omega)\mathcal{O}_+^{in}(\Omega') \rangle = N_\gamma^2 \frac{\mu}{2} \delta(\Omega, \Omega') \quad (4.26)$$

and

$$\begin{aligned} \langle \gamma | \mathcal{O}_+^{in}(\Omega)\mathcal{O}_-^{in}(\Omega') | \gamma \rangle &= N_\gamma^2 e^{\gamma+\gamma*} \langle \mathcal{O}_+^{out}(\Omega)\mathcal{O}_-^{out}(\Omega') \rangle = \\ &= N_\gamma^2 e^{\gamma+\gamma*} \mu^2 \sinh^2 \pi\mu \int d\Omega'' d\Omega''' \Delta_-(\Omega, \Omega'') \frac{\mu}{2} \delta(\Omega'', \Omega''') \Delta_+(\Omega, \Omega''') = \frac{\mu}{2} N_\gamma^2 e^{\gamma+\gamma*} \delta(\Omega, \Omega') . \end{aligned} \quad (4.27)$$

These two point functions agree with the correlators that were calculated in [29, 30] from slightly different approach. We have seen that these correlators can be easily calculated using the conjecture that the states in the dual CFT correspond to the states of massive scalar field in de Sitter space at \mathcal{I}^- .

5. Conclusion

In this paper we tried to present our point of view on the nature of dS/CFT correspondence. We studied the relation between free massive scalar field in the bulk of de Sitter space and the dual Euclidean CFT living in the past infinity \mathcal{I}^- . Using the definition of the inner product in the Hilbert space \mathcal{H}_i of initial states of massive scalar field in de Sitter space we were able to reconstruct all two point functions of dual CFT operators in arbitrary vacuum state. Let us express once again the main idea of our approach. Following arguments given in [41] we consider operators in dual CFT living on \mathcal{I}^- that define states in the Hilbert space \mathcal{H}_i of initial states of quantum gravity in de Sitter space. Using this conjecture we can from the known inner product and the structure of the Hilbert space of initial states on \mathcal{I}^- obtain all correlators in the dual CFT and define adjoint operation. We have seen that application of this idea gives correlators in the dual CFT that have the same form as the two point functions that were calculated in [29, 30] from the equivalence of partition function of CFT on \mathcal{I}^- and S-matrix elements in the bulk.

It is clear that the same analysis, as was presented in previous sections, could be performed for CFT defined on \mathcal{I}^+ in the sense that this theory gives an exact description of the Hilbert space H_f of the final states of massive scalar field at past infinity. Of course in the case of the example studied in this paper these two Hilbert spaces are identical and hence corresponding CFT are the same. For that reason we considered the CFT defined on \mathcal{I}^- as the fundamental one. However it is clear that one single CFT cannot contain information about dynamical evolution of the bulk quantum gravity which, in our personal point of view, should have its CFT description in terms of renormalization group flow [33] that maps CFT operators on \mathcal{I}^+ corresponding to UV limit of the theory to the CFT operators on \mathcal{I}^- corresponding to IR limit.

The extension of this approach is natural. The first possibility is to apply this analysis to the case of arbitrary μ as in [31, 34]. It would be also nice to analyse massless bulk scalar field with its potential application to the cosmology [62, 63, 64, 65]. Very interesting analysis of this problem from the point of view of dS/CFT was given recently in [61]. We would like also study the relation between CFT and quantum field theory in Elliptic de Sitter space [32] and the relation between CFT and quantum field theory in the space-times that approach their de Sitter phase in the asymptotic past or future only.

Even if we know that this paper gives only modest contribution to the question of the quantum gravity in de Sitter space and its relation to the string theory we believe that this small contribution could be helpful for further study of dS/CFT correspondence.

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References

- [1] A. Balbi *et al.*, “*Constraints on cosmological parameters from MAXIMA-1*,” *Astrophys. J.* **545**, L1 (2000) [Erratum-ibid. **558**, L145 (2001)] [arXiv:astro-ph/0005124].
Astrophys. J. **558**, L145 (19yy).
- [2] R. Stompor *et al.*, “*Cosmological implications of the MAXIMA-I high resolution Cosmic Microwave Background anisotropy measurement*,” *Astrophys. J.* **561**, L7 (2001) [arXiv:astro-ph/0105062].
- [3] S. Perlmutter *et al.* [Supernova Cosmology Project Collaboration], “*Measurements of the Cosmological Parameters Ω and Λ from the First 7 Supernovae at $z \leq 0.35$* ,” *Astrophys. J.* **483**, 565 (1997) [arXiv:astro-ph/9608192].
- [4] A. G. Riess *et al.* [Supernova Search Team Collaboration], “*Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant*,” *Astron. J.* **116**, 1009 (1998) [arXiv:astro-ph/9805201].
- [5] J. R. Primack, “*Status of cold dark matter cosmology*,” arXiv:astro-ph/0205391.
- [6] T. Banks, “*Cosmological Breaking Of Supersymmetry?*,” *Int. J. Mod. Phys. A* **16**, 910 (2001).
- [7] T. Banks and W. Fischler, “*M-theory observables for cosmological space-times*,” arXiv:hep-th/0102077.
- [8] T. Banks, “*Cosmological breaking of supersymmetry or little Lambda goes back to the future. II*,” arXiv:hep-th/0007146.
- [9] L. Smolin, “*Quantum gravity with a positive cosmological constant*,” arXiv:hep-th/0209079.
- [10] T. Banks, W. Fischler and S. Paban, “*Recurrent nightmares?: Measurement theory in de Sitter space*,” *JHEP* **0212** (2002) 062 [arXiv:hep-th/0210160].
- [11] N. Goheer, M. Kleban and L. Susskind, “*The trouble with de Sitter space*,” arXiv:hep-th/0212209.
- [12] U. H. Danielsson, D. Domert and M. Olsson, “*Miracles and complementarity in de Sitter space*,” arXiv:hep-th/0210198.
- [13] A. Strominger, “*The dS/CFT correspondence*,” *JHEP* **0110**, 034 (2001) [arXiv:hep-th/0106113].
- [14] F. L. Lin and Y. S. Wu, “*Near-horizon Virasoro symmetry and the entropy of de Sitter space in any dimension*,” *Phys. Lett. B* **453**, 222 (1999) [arXiv:hep-th/9901147].
- [15] D. Klemm, “*Some aspects of the de Sitter/CFT correspondence*,” *Nucl. Phys. B* **625**, 295 (2002) [arXiv:hep-th/0106247].

- [16] B. McInnes, “*Exploring the similarities of the dS/CFT and AdS/CFT correspondences*,” Nucl. Phys. B **627**, 311 (2002) [arXiv:hep-th/0110062].
- [17] V. Balasubramanian, J. de Boer and D. Minic, “*Mass, entropy and holography in asymptotically de Sitter spaces*,” Phys. Rev. D **65**, 123508 (2002) [arXiv:hep-th/0110108].
- [18] B. G. Carneiro da Cunha, “*Three-dimensional de Sitter gravity and the correspondence*,” Phys. Rev. D **65**, 104025 (2002) [arXiv:hep-th/0110169].
- [19] R. G. Cai, “*Cardy-Verlinde formula and asymptotically de Sitter spaces*,” Phys. Lett. B **525**, 331 (2002) [arXiv:hep-th/0111093].
- [20] S. Nojiri and S. D. Odintsov, “*Wilson loop and dS/CFT correspondence*,” Phys. Lett. B **528**, 169 (2002) [arXiv:hep-th/0112152].
- [21] S. Hawking, J. M. Maldacena and A. Strominger, “*DeSitter entropy, quantum entanglement and AdS/CFT* ,” JHEP **0105**, 001 (2001) [arXiv:hep-th/0002145].
- [22] R. Bousso, “*Bekenstein bounds in de Sitter and flat space*,” JHEP **0104**, 035 (2001) [arXiv:hep-th/0012052].
- [23] V. Balasubramanian, P. Horava and D. Minic, “*Deconstructing de Sitter*,” JHEP **0105**, 043 (2001) [arXiv:hep-th/0103171].
- [24] M. Li, “*Matrix model for de Sitter*,” JHEP **0204**, 005 (2002) [arXiv:hep-th/0106184].
- [25] E. Halyo, “*De Sitter entropy and strings*,” arXiv:hep-th/0107169.
- [26] C. M. Hull, “*De Sitter space in supergravity and M theory*,” JHEP **0111**, 012 (2001) [arXiv:hep-th/0109213].
- [27] U. H. Danielsson, “*A black hole hologram in de Sitter space*,” JHEP **0203**, 020 (2002) [arXiv:hep-th/0110265].
- [28] S. Ogushi, “*Holographic entropy on the brane in de Sitter Schwarzschild space*,” Mod. Phys. Lett. A **17**, 51 (2002) [arXiv:hep-th/0111008].
- [29] R. Bousso, A. Maloney and A. Strominger, “*Conformal vacua and entropy in de Sitter space*,” Phys. Rev. D **65**, 104039 (2002) [arXiv:hep-th/0112218].
- [30] M. Spradlin and A. Volovich, “*Vacuum states and the S-matrix in dS/CFT* ,” Phys. Rev. D **65** (2002) 104037 [arXiv:hep-th/0112223].
- [31] F. Leblond, D. Marolf and R. C. Myers, “*Tall tales from de Sitter space. II: Field theory dualities*,” arXiv:hep-th/0211025.
- [32] M. K. Parikh, I. Savonije and E. Verlinde, “*Elliptic de Sitter space: $dS/Z(2)$* ,” arXiv:hep-th/0209120.
- [33] A. Strominger, “*Inflation and the dS/CFT correspondence*,” JHEP **0111**, 049 (2001) [arXiv:hep-th/0110087].

- [34] A. J. Tolley and N. Turok, “*Quantisation of the massless minimally coupled scalar field and the dS/CFT correspondence*,” arXiv:hep-th/0108119.
- [35] V. Balasubramanian, J. de Boer and D. Minic, “*Exploring de Sitter space and holography*,” arXiv:hep-th/0207245.
- [36] D. Klemm and L. Vanzo, “*De Sitter gravity and Liouville theory*,” JHEP **0204**, 030 (2002) [arXiv:hep-th/0203268].
- [37] F. Larsen, J. P. van der Schaar and R. G. Leigh, “*de Sitter holography and the cosmic microwave background*,” JHEP **0204**, 047 (2002) [arXiv:hep-th/0202127].
- [38] F. Leblond, D. Marolf and R. C. Myers, “*Tall tales from de Sitter space. I: Renormalization group flows*,” JHEP **0206**, 052 (2002) [arXiv:hep-th/0202094].
- [39] S. R. Das, “*Thermalities in de Sitter and holography*,” Phys. Rev. D **66**, 025018 (2002) [arXiv:hep-th/0202008].
- [40] D. Kabat and G. Lifschytz, “*De Sitter entropy from conformal field theory*,” JHEP **0204**, 019 (2002) [arXiv:hep-th/0203083].
- [41] E. Witten, “*Quantum gravity in de Sitter space*,” arXiv:hep-th/0106109.
- [42] J. M. Maldacena, “*The large N limit of superconformal field theories and supergravity*,” Adv. Theor. Math. Phys. **2**, 231 (1998) [Int. J. Theor. Phys. **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [43] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “*Gauge theory correlators from non-critical string theory*,” Phys. Lett. B **428**, 105 (1998) [arXiv:hep-th/9802109].
- [44] E. Witten, “*Anti-de Sitter space and holography*,” Adv. Theor. Math. Phys. **2**, 253 (1998) [arXiv:hep-th/9802150].
- [45] N. A. Chernikov and E. A. Tagirov, “*Quantum Theory Of Scalar Fields In De Sitter Space-Time*,” Annales Poincare Phys. Theor. A **9**, 109 (1968).
- [46] E. A. Tagirov, “*Consequences Of Field Quantization In De Sitter Type Cosmological Models*,” Annals Phys. **76**, 561 (1973).
- [47] B. Allen, “*Vacuum States In De Sitter Space*,” Phys. Rev. D **32**, 3136 (1985).
- [48] E. Mottola, “*Particle Creation In De Sitter Space*,” Phys. Rev. D **31**, 754 (1985).
- [49] K. Kirsten and J. Garriga, “*Massless Minimally Coupled Fields In De Sitter Space: $O(4)$ Symmetric States Versus De Sitter Invariant Vacuum*,” Phys. Rev. D **48**, 567 (1993) [arXiv:gr-qc/9305013].
- [50] N. D. Birrell and P. C. Davies, “*Quantum Fields In Curved Space*,” SPIRES entry .
- [51] R. M. Wald, “*Quantum Field Theory In Curved Space-Time And Black Hole Thermodynamics*,” SPIRES entry

- [52] M. Spradlin, A. Strominger and A. Volovich, “*Les Houches lectures on de Sitter space*,” arXiv:hep-th/0110007.
- [53] Y. b. Kim, C. Y. Oh and N. Park, arXiv:hep-th/0212326.
- [54] N. Kaloper, M. Kleban, A. Lawrence, S. Shenker and L. Susskind, “*Initial conditions for inflation*,” JHEP **0211**, 037 (2002) [arXiv:hep-th/0209231].
- [55] J. Martin and R. H. Brandenberger, “*The trans-Planckian problem of inflationary cosmology*,” Phys. Rev. D **63**, 123501 (2001) [arXiv:hep-th/0005209].
- [56] R. Easther, B. R. Greene, W. H. Kinney and G. Shiu, “*Imprints of short distance physics on inflationary cosmology*,” arXiv:hep-th/0110226.
- [57] N. Kaloper, M. Kleban, A. E. Lawrence and S. Shenker, “*Signatures of short distance physics in the cosmic microwave background*,” Phys. Rev. D **66**, 123510 (2002) [arXiv:hep-th/0201158].
- [58] R. H. Brandenberger and J. Martin, “*On signatures of short distance physics in the cosmic microwave background*,” Int. J. Mod. Phys. A **17**, 3663 (2002) [arXiv:hep-th/0202142].
- [59] T. Banks and L. Mannelli, “*De Sitter vacua, renormalization and locality*,” arXiv:hep-th/0209113.
- [60] M. B. Einhorn and F. Larsen, “*Interacting quantum field theory in de Sitter vacua*,” arXiv:hep-th/0209159.
- [61] J. Maldacena, “*Non-Gaussian features of primordial fluctuations in single field inflationary models*,” arXiv:astro-ph/0210603.
- [62] U. H. Danielsson, “*Inflation, holography and the choice of vacuum in de Sitter space*,” JHEP **0207** (2002) 040 [arXiv:hep-th/0205227].
- [63] L. Bergstrom and U. H. Danielsson, “*Can MAP and Planck map Planck physics?*,” JHEP **0212**, 038 (2002) [arXiv:hep-th/0211006].
- [64] U. H. Danielsson, “*A note on inflation and transplanckian physics*,” Phys. Rev. D **66**, 023511 (2002) [arXiv:hep-th/0203198].
- [65] U. H. Danielsson, “*On the consistency of de Sitter vacua*,” JHEP **0212**, 025 (2002) [arXiv:hep-th/0210058].
- [66] D. V. Long and G. M. Shore, “*The Schrodinger Wave Functional and Vacuum States in Curved Spacetime*,” Nucl. Phys. B **530** (1998) 247 [arXiv:hep-th/9605004].